24. Proposed by C. W. M. BLACK, A. M., Department of Mathematics, Wesleyan Academy, Wilbraham, Massachusetts.

At the President's reception, the people are admitted at 12½ P. M.; but the *line* in front of the gate, begins to form at 11 A. M. By the time the doors are opened, there are in line 5400 citizens, who have gathered at a rate per second proportional to the time after 11 A. M. The President receives the citizens at the uniform rate of 15 per minute. At what time after 11 A. M. should a citizen *join the line*, in order that he may be *delayed the least* by the reception?

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland, and the PROPOSER.

Let $T_1=5400$, seconds = the *line-formation* time; $P_1=5400$, persons = the number of citizens in line at the expiration of the *first* second; x = the number of seconds after 11 A. M. a citizen should join the line, in order to suffer the *least delay;* and U = the number of seconds a citizen joining the line, would be delayed by the reception. The number of citizens joining the line, per second, forms during the time T, an arithmetical series in which $a=P_1$, $n=T$, $d=P_1$, $l=a+(n-1)d=TP_1$, and $s=\frac{1}{2}n(a+l)=\frac{1}{2}T(1+T)P_1=P$. $\therefore$ $P_1=2P/T(T+1)$; and consequently, the number of citizens who joined the line during the xth second, is $P_1x=2Px/T(T+1)$. Hence the number of citizens in line at the expiration of the xth second, is

$$N=\frac{x(P_1+P_1x)}{2},=\left(\frac{x(x+1)}{T(T+1)}\right)P\ldots.(1).$$

Let R = the number of citizens the President receives per *minute*; then it will take 60 (N/R) *seconds* to receive N citizens. Since the citizens who will actually suffer the least delay has already been delayed $(T-x)$ seconds (waiting in line), the expression for the *total delay*, in seconds, becomes

$$U=(T-x)+\frac{60}{R}\left(\frac{x(x+1)}{T(T+1)}\right)P=\text{a maximum}\ldots\ (2).$$

$$\therefore\ \frac{dU}{dx}=-1+\frac{60}{R}\left(\frac{2x+1}{T(T+1)}\right)P=0\ldots.(3),$$

and $x=\frac{1}{2}\left[\frac{R}{60}\left(\frac{T(T+1)}{P}\right)-1\right],=2024\frac{3}{8}$ seconds; that is, the proper time for joining the line is 33 minutes $44\frac{3}{8}$ seconds after 11 A.M. The total delay occasioned by the reception, is U=1 hour 13 minutes $7\frac{3}{8}$ seconds.

Also solved by *C. E. White and G. B. M. Zerr*

PROBLEMS.

32. Proposed by J. F. W. SCHEFFER, Hagerstown, Maryland.

Suppose it to be possible to perform the passage through the north pole: at what latitude would the maximum distance be saved by a ship sailing on the arc of a great circle instead of a parallel of latitude, the points of departure and destination being 180° apart? Also find the maximum saving.

33. Proposed by WILLIAM SYMMOND, A. M., Professor of Mathematics and Astronomy, Pacific College, Santa Rosa, California.

Show that of all curves of a given length, traced on one plane between two given points, and made to revolve around a common axis situated in that plane, the Catenary generates a minimum area.

MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

12. Proposed by J. F. W. SCHEFFER, A. M. Hagerstown, Maryland.

A horizontal table without weight is supported on three points, A, B, and C. A weight W is laid upon the table, at a point G. If $AG=a$, $BG=b$, $CG=c$, $\angle AGB=\alpha$, $\angle BGC=\beta$, and $\angle CGA=\gamma$, find the pressures upon A, B, and C.

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

The momental equations with respect to AG, BG, and CG, are respectively:

$$P_B\times b\sin(\pi-\alpha)=P_C\times c\sin\beta. \;\therefore\; \frac{P_B}{P_C}=\frac{c\sin\beta}{b\sin\alpha}\ldots.(m).$$

$$P_C\times c\sin(\pi-\beta)=P_A\times a\sin[\gamma-(\pi-\beta)]. \;\therefore\; P_C=\left(\frac{a\sin\alpha}{c\sin\beta}\right)P_A\ldots.(n).$$

$$P_A\times a\sin(\pi-\gamma)=P_B\times b\sin[\alpha-(\pi-\gamma)]. \;\therefore\; P_B=\left(\frac{a\sin\gamma}{b\sin\beta}\right)P_A\ldots.(p).$$

Put $K=\left(\frac{\sin\beta}{a}+\frac{\sin\gamma}{b}+\frac{\sin\alpha}{c}\right)$; then from the equation, $P_A+P_B+P_C=W$, we deduce the following *symmetrical* and elegant results:

$$P_A=\left(\frac{\sin\beta}{aK}\right)W,\; P_B=\left(\frac{\sin\gamma}{bK}\right)W, \text{ and } P_C=\left(\frac{\sin\alpha}{cK}\right)W.$$

Second Solution.

According to the logic of common-sense, why not write

$$P_A=\left(\frac{\triangle BCG}{\triangle ABC}\right)W,\; P_B=\left(\frac{\triangle CAG}{\triangle ABC}\right)W, \text{ and } P_C=\left(\frac{\triangle ABG}{\triangle ABC}\right)W;$$

and then with the heavy artillery of the higher mathematics *successfully defend* our position ?

NOTE:—These two solutions are to take the place of the *first* solution of this problem published in the November MONTHLY.—*F. P. M.*